EVAPORATION OF WATER INTO A LAMINAR STREAM OF AIR AND SUPERHEATED STEAM

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Abstract — A numerical investigation was conducted to study the evaporation of water into a laminar stream of air, humid air and superheated steam. Results were obtained both for variable properties and for constant properties using the one-third rule. It was shown that below a certain temperature of the free stream, named the inversion temperature, water evaporation decreases as the humidity of air increases; and above the inversion temperature, water evaporation increases as the humidity of air increases. The constant-property approximation using the one-third rule yields accurate results for the evaporation rate of water except when the free stream is mostly steam at high temperature.

NOMENCLATURE

- $c_{\rm f}$, friction coefficient;
- c_p , specific heat of fluid [J kg⁻¹ °C⁻¹];
- D, mass diffusion coefficient $[m^2 s^{-1}]$;
- f, dimensionless stream function;
- $h_{\rm fg}$, latent heat of evaporation [J kg⁻¹];
- k, thermal conductivity of fluid $[W m^{-1} \circ C^{-1}];$
- L, length of water surface [m];
- m, mass fraction;
- \dot{m} , mass flux [kg m⁻² s⁻¹];
- M, molecular weight;
- Nu, Nusselt number;
- P, partial pressure [bar];
- *Pr*, Prandtl number, $\mu c_p/k$;
- *Re*, Reynolds number, $\rho u x / \mu$;
- Sc, Schmidt number, $\mu/\rho D$;
- *Sh*, Sherwood number;
- T, temperature [$^{\circ}C$];
- u, velocity component in x direction $[m s^{-1}]$;
- v, velocity component in y direction $[m s^{-1}]$;
- x, horizontal spatial coordinate [m];
- y, vertical spatial coordinate [m].

Greek symbols

- η , independent similarity variable;
- θ , dimensionless temperature of fluid;
- μ , dynamic viscosity of fluid [kg m⁻¹ s⁻¹];
- v, kinematic viscosity of fluid $[m^2 s^{-1}]$;
- ρ , density of fluid [kg m⁻³];
- ψ , stream function [kg m⁻¹ s⁻¹];
- ϕ_{c_p} , ratio of local c_p to $c_{p_{\infty}}$;
- ϕ_k , ratio of local k to k_{∞} ;
- ϕ_{μ} , ratio of local μ to μ_{∞} ;
- ϕ_{ρ} , ratio of local ρ to ρ_{∞} .

Subscripts

- a, air;
- ev, evaporation;
- 0, fluid-water interface;
- w, steam;
- ∞ , free stream.

Superscripts

- *, reference state;
- , average;
- differentiation with respect to η .

INTRODUCTION

DUE TO its widespread applications, such as in drying, film cooling and air conditioning, the subject of evaporation of water into an air stream has received considerable attention. Superheated steam has also been recommended as an attractive drying medium for materials that are not temperature sensitive. Wenzel and White [1] studied the drying of granular solids in superheated steam and reported higher drying rates in superheated steam than in air at the same temperature and mass flow rate. Moreover, for the case of drying with superheated steam, the water removed from the solids during the drying process becomes a part of the drying medium whereas, in air drying, the moist air must ultimately be replaced by fresh air heated to a suitable temperature. Yoshida and Hyõdõ [2] demonstrated that superheated steam can provide an excellent medium for drying food products. As well as allowing a higher evaporation rate as compared to air at the same temperature and mass flow rate, steam is cleaner and there is less oxidation in food, thus reducing the loss in nutritional value during the drying process.

Chu et al. [3] investigated experimentally the evaporation of liquids into their superheated vapors. Three liquids were used, water, 1-butanol and benzene. They compared their results with available data using air as the drying medium. They reported significantly higher rate of evaporation with superheated vapor than with air except when the superheated vapor temperature is relatively close to the saturation temperature. In fact, when the vapor is near its saturation temperature, the evaporation rate should approach zero, whereas evaporation will still occur when air is at the saturation temperature of the liquid. This leads to a critical temperature at which the rate of evaporation into its superheated vapor is equal to the rate of evaporation into air. Chu *et al.* [4] studied the evaporation of water into superheated steam-air mixtures. Again, they reported that superheated steam will help the drying rate when mixed with air.

Yoshida and Hyõdõ [5] also investigated the evaporation of water into air, humid air and superheated steam. Since it is generally accepted that water evaporation decreases as the humidity of air increases, they were particularly concerned with the critical (inversion) temperature mentioned above. They showed experimentally the existence of an inversion temperature. Above the inversion temperature, water evaporation actually increases as the humidity of air increases and the evaporation rate is the highest for pure superheated steam. Trommelen and Crosby [6] investigated the evaporation of water from drops of nine different materials. The drops were approximately 1.5 mm in diameter and both superheated steam and dry air were employed as drying media. They also reported the possible existence of an inversion temperature.

The objective of the present work is two fold. First, the physical reason for the existence of the inversion temperature is investigated. The situation considered here is a laminar boundary-layer flow of air, steam or a mixture of the two over a water surface. Laminar flow is chosen so that analytical expressions for the evaporation rate can be obtained. Thus, the physical explanation for the above phenomenon can be focused upon. In the present problem, the temperature profile across the boundary layer is very steep and the thermophysical properties can change significantly. Numerical results are obtained both for variable properties and for constant properties using a reference state, namely, the one-third rule. It is well established that the one-third rule is suitable for convective heat transfer [7] and for simultaneous heat and mass transfer of droplets [8]. The second objective of the present study is to verify the suitability of the one-third rule for the present situation.

ANALYSIS

Governing equations

The schematic of the present problem is shown in Fig. 1. Steady-state evaporation of water into a laminar stream of air, humid air or superheated steam is considered. The pressure of the stream is atmospheric



FIG. 1. The schematic of the mathematical model.

(1.013 bar). It is assumed that the water-fluid interface is stationary and no energy is supplied from beneath the interface. Hence, the energy required for the evaporation of water must come from the stream itself. Furthermore, it is assumed that the gradients of the velocity, u, the mass concentrations, m_a and m_w , and the temperature, T, in the flow direction are much smaller than those in the direction transverse to the flow so that the boundary-layer approximations are valid.

The governing equations are

mass
$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0,$$
 (1)

momentum
$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = \frac{\partial}{\partial y} \left(\mu \frac{\partial u}{\partial y} \right),$$
 (2)

species
$$\rho u \frac{\partial m_{a}}{\partial x} + \rho v \frac{\partial m_{a}}{\partial y} = \frac{\partial}{\partial y} \left(\rho D \frac{\partial m_{a}}{\partial y} \right),$$
 (3)

energy $\rho c_p u \frac{\partial T}{\partial x} + \rho c_p v \frac{\partial T}{\partial y}$

$$= \frac{\partial}{\partial y} \left(k \frac{\partial T}{\partial y} \right) + \rho D(c_{pa} - c_{pw}) \frac{\partial m_a}{\partial y} \frac{\partial T}{\partial y}.$$
 (4)

The boundary conditions are

for
$$y > 0$$
, at $x = 0$, $u = u_{\infty}, v = 0$,
 $T = T_{x},$ (5)
 $m_{a} = m_{a\infty},$

for x > 0, at y = 0, $u = 0, v = v_0(x)$,

$$T = T_0(x), \tag{6}$$

 $m_{\rm a}=m_{\rm a0}(x),$

for x > 0, as $y \to \infty$, $u = u_x, v = 0$,

$$T = T_x, \tag{7}$$

$$m_{\rm a}=m_{\rm ax}$$
.

At the interface, y = 0, the fluid velocity, $v_0(x)$, and the temperature, $T_0(x)$, can be obtained by the following relations:

zero net mass flux of air at interface,

at
$$y = 0$$
, $m_{a}\rho v - \rho D \frac{\partial m_{a}}{\partial y} = 0$, (8)

energy balance at interface,

$$t \quad y = 0, \quad k \frac{\partial T}{\partial y} = \rho v h_{fg}. \tag{9}$$

Also, the mass concentration of air at the interface can be obtained by assuming ideal-gas mixtures for the fluid,

$$m_{a0} = \frac{P_{a0}M_a}{P_{a0}M_a + P_{w0}M_w}$$
(10)

where M_a , M_w are the molecular weights of air and steam, respectively; P_{a0} , P_{w0} are the partial pressures of air and steam at the interface, respectively. The partial pressure of steam at the interface, P_{w0} , is the vapor pressure of steam at T_0 .

For the case when the laminar stream is composed of pure superheated steam, the species equation and the associated boundary conditions become irrelevant. In addition, the term involving the mass concentration of air in equation (4) should be ignored.

Similarity solutions

For laminar flows of air or air-stream mixtures, since there is no energy supplied from below the fluid-water interface, the interfacial temperature, $T_0(x)$, should be equal to the wet bulb temperature of the free stream. Hence, it is reasonable to assume $T_0(x)$ to be constant along the water surface. For the case when the free stream is pure superheated steam, the interfacial temperature should be equal to the saturation temperature of steam at one atmosphere (100°C).

The invariance of the interfacial temperature, T_0 , with x allows great simplification of the solution procedure. It can be shown that similarity solutions for the present problems exist [9]. First, the continuity equation can be eliminated by using the stream function as a dependent variable,

$$\rho u = \frac{\partial \psi}{\partial y}, \quad \rho v = -\frac{\partial \psi}{\partial x}.$$
(11)

By introducing the following variables:

$$\theta = (T - T_0)/(T_{\infty} - T_0),$$

$$\eta = y \left(\frac{u_{\infty}}{v_{\infty} x}\right)^{1/2},$$
(12)

$$\psi = (\rho_{\infty} \mu_{\infty} u_{\infty} x)^{1/2} f,$$

the governing equations, equations (2)-(4), become

$$[\phi_{\mu}(f'/\phi_{\rho})']' + \frac{1}{2}f(f'/\phi_{\rho})' = 0, \qquad (13)$$

$$\left[\frac{\phi_{\mu}}{Sc}m'_{a}\right]' + \frac{1}{2}fm'_{a} = 0, \qquad (14)$$

 $(\phi_k \theta')' + \frac{1}{2} Pr_{\infty} \phi_{c_p} f \theta'$

$$+\phi_{c_p}\phi_{\mu}(c_{pa}-c_{pw})Pr_{\infty}m'_{a}\theta'/(c_pSc)=0.$$
 (15)

The symbol ϕ_k is defined as the ratio of the local k to the k at free stream. A similar definition holds for ϕ_{c_p} , ϕ_{μ} and ϕ_p . These are functions of θ and m_a and are therefore dependent on η implicitly. The dependent variables f, m_a and θ are functions of η only. The velocities u and v are given by

$$u = u_{\infty} f' / \phi_{\rho}, \qquad (16)$$

$$v = \frac{1}{2} \left(\frac{u_{\infty} v_{\infty}}{x} \right)^{1/2} (\eta f' - f) / \phi_{\rho}.$$
 (17)

The boundary conditions for f, m_a and θ are

$$\eta = 0, \quad f = f_0,$$

$$f' = 0,$$

$$m_a = m_{a0},$$

$$\theta = 0,$$

(18)

as
$$\eta \to \infty$$
, $f'/\phi_{\rho} = 1$,
 $m_{a} = m_{a\infty}$, (19)
 $\theta = 1$

The condition of zero net mass flux of air and the energy balance at the interface, equations (8) and (9), can be rewritten as

at
$$\eta = 0$$
, $\phi_{\mu}m'_{a} + \frac{Sc}{2}fm_{a} = 0$, (20)

at
$$\eta = 0$$
, $\phi_k \theta' = -\frac{f}{2} Pr_\infty \frac{h_{fg}}{c_{p_\infty}(T_\infty - T_0)}$ (21)

where h_{fg} is the latent heat of evaporation at the interfacial temperature, T_0 .

Again, as discussed in the previous section, when the free stream is composed of pure superheated steam, the species equation and the associated boundary conditions will simply be ignored.

Reference state—the one-third rule

The above analysis is repeated by evaluating the thermophysical properties ρ , μ , k, c_p and D at a reference state with temperature T^* and air mass concentration m_a^* ,

$$T^* = T_0 + \frac{1}{3}(T_{\infty} - T_0),$$

$$m_a^* = m_{a0} + \frac{1}{3}(m_{a\infty} - m_{a0}).$$
(22)

Using these 'constant' properties, the similarity variable η , the stream function ψ , and the velocities u, v are redefined as

$$\eta = y \left(\frac{u_{\infty}}{v_X}\right)^{1/2} ,$$

$$\psi = (\rho \mu u_{\infty} x)^{1/2} f,$$

$$u = u_{\infty} f',$$

$$v = \frac{1}{2} \left(\frac{u_{\infty} v}{x}\right)^{1/2} (\eta f' - f).$$
(23)

The similarity equations, equations (13)-(15), become

$$f''' + \frac{1}{2}ff'' = 0, (24)$$

$$m_a'' + \frac{Sc}{2}fm_a' = 0,$$
 (25)

$$\theta'' + \frac{1}{2} Prf\theta' + \frac{c_{pa} - c_{pw}}{c_p} \frac{Pr}{Sc} m'_a \theta' = 0.$$
 (26)

The boundary conditions become

at
$$\eta = 0$$
, $f = f_0$,
 $f' = 0$,
 $m_a = m_{a0}$,
 $\theta = 0$,
as $\eta \to \infty$, $f' = 1$,
 $m_a = m_{a\infty}$, (28)
 $\theta = 1$.

The two conditions at the interface become

at
$$\eta = 0$$
, $m'_{a} + \frac{Sc}{2}fm_{a} = 0$, (29)

at
$$\eta = 0$$
, $\theta' = -\frac{f}{2} Pr \frac{h_{fg}}{c_p(T_{\infty} - T_0)}$ (30)

where $h_{\rm fg}$ is evaluated at the interfacial temperature.

Thermophysical properties

The properties of air, superheated steam and their mixtures are obtained from the following sources:

$\rho_{\rm a}, \rho_{\rm w},$	ideal gas relations;
ρ ,	$= \rho_{\mathbf{a}} + \rho_{\mathbf{w}};$
μ_{a} ,	ref. [10], p. 23;
μ_w ,	ref. [11];
μ,	ref. [12];
$k_{\rm a}, k_{\rm w},$	ref. [13];
<i>k</i> ,	ref. [10], p. 257;
C_{pa} ,	ref. [14];
c_{pw} ,	ref. [13];
c_p ,	$= m_{a}c_{pa} + m_{w}c_{pw};$
Ď,	ref. [10], p. 511.

Numerical solution procedure

The similarity equations (13)-(15), subject to conditions (18)-(21), and equations (24)-(26), subject to conditions (27)-(30), are solved numerically by the quasilinearization method. This method has been successfully applied to two-point boundary-value problems [15].

For the cases when the free stream is composed of either air or humid air, the calculation starts with an initial guess of the interfacial temperature, T_0 , and an assumed profile for each dependent variable, f, m_a and θ . Based on the guessed T_0 , m_{a0} can be determined from equation (10). For the variable-property case, the value of f_0 is then updated by the condition of zero net mass flux of air, equation (20). Next, the similarity solutions (13)-(15) are solved to obtain a new profile for f, m_a and θ . These profiles are used to update f_0 again, using equation (20). This process is repeated until the percentage change of all the dependent variables between two successive iterations is less than 10^{-5} . The converged results are used in the energy balance equation (21), to check if the assumed T_0 is correct. If not, another value for T_0 will be used and the above procedure is repeated.

The calculation procedure is simpler for the case when the free stream is composed of pure superheated steam. This is because the interfacial temperature is equal to 100°C. The species equation and the associated boundary conditions, including equation (20), are ignored, and f_0 is updated with the energy balance at the interface, equation (21). Again, the iterative process stops when the percentage change of f and θ between two successive iterations is less than 10^{-5} .

The calculation procedure for cases using the onethird rule approximation is identical to that described above.

RESULTS AND DISCUSSION

In previous experimental investigations, comparisons of the rates of evaporation of water into air, humid air or superheated steam were made with the same mass flux and temperature of the free stream. The inversion temperature was obtained based on these comparisons. In the present work, the evaporation rates, \dot{m}_{ev} , are also compared in the same manner.

The average \bar{m}_{ev} for a water surface of length L is defined as

$$\bar{m}_{ev} = \frac{1}{L} \int_0^L \vec{m}_{ev}(x) \, \mathrm{d}x.$$
 (31)

From equations (17) and (23), it can be shown that.

$$\bar{m}_{\rm ev} = -f_0 \mu_{\infty}^{1/2} \left(\frac{\dot{m}_{\infty}}{L}\right)^{1/2}, \text{ (variable properties)} \qquad (32)$$

$$\vec{m}_{\rm ev} = -f_0 \left(\frac{\rho}{\rho_{\perp}} \mu\right)^{1/2} \left(\frac{\dot{m}_{\infty}}{L}\right)^{1/2} \text{ (one-third rule)} \quad (33)$$

where \dot{m}_{α} is equal to $\rho_{\alpha}u_{\alpha}$, the mass flux of the free stream.

Most of the calculations performed are for the variable-property case. Some calculations are also presented using the one-third rule. These latter calculations are used merely to compare with those of variable properties to demonstrate whether the onethird rule is suitable for the present situation.

In Fig. 2, the water evaporation rates for the same free stream mass flux and length of the water surface are compared for three different compositions of the free stream. The three compositions are pure air, pure superheated steam and a mixture of equal amount by mass of air and superheated steam. At the lower free stream temperatures, water does evaporate faster in air than in humid air and in superheated steam. However, the trend is reversed for the higher free stream temperatures, namely, water evaporation actually increases as the humidity of air increases and the evaporation rate is the highest for pure superheated steam. Another observation needing to be noted is that the inversion temperature is not unique. Specifically, the curves representing the evaporation rates for the three cases intersect at three different temperatures. The inversion temperature may be defined as the intersection between the curves for pure superheated steam and for pure air. The intersecting temperatures between pure steam and any other humid air compositions are always lower than the inversion temperature defined above.

In Fig. 3, the evaporation rates are plotted as a function of the free stream air mass concentration for three different temperatures. At $T_{\chi} = 350^{\circ}$ C, the evaporation rate increases monotonically as $m_{a,\tau}$ decreases. At $T_{\alpha} = 300^{\circ}$ C, the evaporation rate is fairly constant as $m_{a,\alpha}$ decreases from 1 to 0.5, and it begins to increase when $m_{a,\alpha}$ drops to below 0.5. At $T_{\alpha} = 200^{\circ}$ C, the opposite trend holds, the evaporation rate drops when $m_{a,\alpha}$ is reduced from 1 to 0.7 and stays almost constant for lower values of $m_{a,\alpha}$.



FIG. 2. The evaporation rates for various free stream fluids.

To help explain the existence of the inversion temperature, the variable-property results are given in detail in Table 1. Also, in Fig. 4, the interfacial temperature T_0 is plotted versus the free stream mass concentration for various free stream temperatures. It can be seen that the presence of air depresses T_0 from 100°C to the wet bulb temperature.

The heat transfer from the free stream to the water surface can be written as

$$Q = \bar{h}(T_{\infty} - T_0) \tag{34}$$

where \bar{h} is the heat transfer coefficient. For low T_{∞} , the depression of T_0 due to air-water vapor diffusion causes

a significant percentage increase in the temperature potential $T_{\infty} - T_0$ compared to the temperature potential for the pure steam case. This explains the generally accepted fact that water evaporates faster in air than in moist air. However, if T_{∞} is large, the percentage change in the temperature potential is not significant when T_0 is depressed.

The other quantity that governs the heat transfer is the heat transfer coefficients, h. The h depends on the Reynolds number, the Prandtl number and the thermal conductivity approximately in the following manner:

$$\bar{h} = C \, R e^{1/2} \, P r^a k / L \tag{35}$$



FIG. 3. The evaporation rates vs mass fractions of air in the free stream.

Table 1.	Results (from	variable-pr	onerty	solutions
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$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	1.000
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	1.809
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	0.6516
200 1.0 5.250 2.541 0.942 44.2 3.130 2.738 0.75 4.706 2.350 0.709 76.1 3.159 2.870 0.5 4.813 2.099 0.472 88.1 3.143 2.944 0.25 5.058 1.849 0.235 95.2 3.114 2.988 0.0 5.442 1.619 0.0 100.0 3.074 3.011 300 1.0 7.883 3.077 0.909 52.6 2.986 2.781 0.75 8.212 2.743 0.678 78.2 3.027 2.782 0.5 8.900 2.460 0.448 89.0 3.000 2.787	
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	2.472
0.5 4.813 2.099 0.472 88.1 3.143 2.944 0.25 5.058 1.849 0.235 95.2 3.114 2.988 0.0 5.442 1.619 0.0 100.0 3.074 3.011 300 1.0 7.883 3.077 0.909 52.6 2.986 2.781 0.75 8.212 2.743 0.678 78.2 3.027 2.782 0.5 8.900 2.460 0.448 89.0 3.000 2.797	1.668
0.25 5.058 1.849 0.235 95.2 3.114 2.988 0.0 5.442 1.619 0.0 100.0 3.074 3.011 300 1.0 7.883 3.077 0.909 52.6 2.986 2.781 0.75 8.212 2.743 0.678 78.2 3.027 2.782 0.5 8.900 2.460 0.448 89.0 3.000 2.797	1.135
0.0 5.442 1.619 0.0 100.0 3.074 3.011 300 1.0 7.883 3.077 0.909 52.6 2.986 2.781 0.75 8.212 2.743 0.678 78.2 3.027 2.782 0.5 8.200 2.460 0.448 89.0 3.000 2.797	0.5955
300 1.0 7.883 3.077 0.909 52.6 2.986 2.781 0.75 8.212 2.743 0.678 78.2 3.027 2.782 0.5 8.200 2.460 0.448 89.0 3.000 2.797	
0.75 8.212 2.743 0.678 78.2 3.027 2.782	3.584
0.5 8,000 2,460 0,448 80.0 3,000 2,707	2.782
0.5 0.909 2.400 0.440 09.0 3.000 2.191	1.994
0.25 9.707 2.224 0.222 95.5 2.945 2.821	1.076
0.0 10.54 2.026 0.0 100.0 2.876 2.853	
350 1.0 9.170 3.377 0.893 56.0 2.903 2.826	4.092
0.75 10.03 2.945 0.661 79.2 2.951 2.758	3.317
0.5 10.92 2.637 0.436 89.4 2.927 2.750	2.379
0.25 11.97 2.407 0.215 95.7 2.864 2.765	1.286
0.0 13.06 2.230 0.0 100.0 2.782 2.798	
500 1.0 12.48 4.466 0.843 63.5 2.635 3.080	5.259
0.5 17.42 3.157 0.397 90.7 2.686 2.662	3.453
0.0 20.80 2.839 0.0 100.0 2.510 2.690	

where C is a numerical constant which depends on the evaporation rate, a is approximately equal to 0.33 for low evaporation rate (when $T_{\infty} - T_0$ is of the order of a few hundred °C or less).

The Prandtl number of steam is approximately unity, whereas the Pr for air is about 0.7. The Prandtl number effect gives a 12% heating advantage to steam over air. Steam has a smaller dynamic viscosity as compared to air. Hence, for the same mass flux, steam can have a Reover 60% higher than that of air. This effect will give a 30% heating advantage to steam over air. These advantages are compensated by the higher thermal conductivity of air compared to that of steam (as much as 25% higher for air than for steam). Another factor that contributes favorably to the evaporation rate of water into steam is that the interfacial temperature is higher, and the latent heat of evaporation at 100° C is lower than that at a lower wet bulb temperature.

At high free stream temperature, the effect of the interfacial temperature depression due to the presence



FIG. 4. The water surface temperatures for various free stream conditions.

of air is not as significant as the factors that affect the heat transfer coefficient. Thus, at high T_{∞} , the evaporation rate of water actually increases as the free stream steam mass concentration increases.

Other useful information that can be obtained from Table 1 are the friction coefficient, the Nusselt number and the Sherwood number.

Friction coefficient, $c_{\rm f}$.

$$\frac{c_{\rm f}}{2} = \left[\mu \frac{\partial u}{\partial y} \right]_0 / \rho_\infty u_\infty^2$$
$$= \left[\phi_\mu (f'/\phi_\rho)' \right]_0 R e_\infty^{-1/2}. \tag{36}$$

Nusselt number, Nu.

$$Nu = -\left[k\frac{\partial T}{\partial y}\right]_{0} x/[k_{\infty}(T_{0} - T_{\infty})]$$
$$= \left[\phi_{k}\theta'\right]_{0} Re_{\infty}^{1/2}.$$
 (37)

Sherwood number, Sh.

$$Sh = -\left[\rho D \frac{\partial m_{\mathbf{w}}}{\partial y}\right]_{0} x / \left[\rho_{\infty} D_{\infty} (m_{\mathbf{w}0} - m_{\mathbf{w}\infty})\right]$$
$$= \left[\phi_{\mu} m_{\mathbf{a}}' Sc\right]_{0} Re_{\infty}^{1/2} Sc_{\infty} / (m_{\mathbf{a}\infty} - m_{\mathbf{a}0}). \tag{38}$$

Next, the present results are compared with previous experimental investigations. According to the definition of the inversion temperature, this temperature is about 250°C. Both Chu *et al.* [4] and Yoshida and Hyödõ [5] reported a much lower inversion temperature of approximately 170°C. However, their flow geometries were different and also their flows were turbulent. The difference in the inversion temperatures is probably due to the following reason. For the same mass flux, steam has a higher Reynolds number than air. Since the heat transfer coefficient depends more on the Reynolds number for turbulent flows than for laminar flows, hence, for turbulent flows, the increase in h for the steam case will overcome the depression of T_0 at a lower free stream temperature. For evaporation of water from drops, Trommelen and Crosby [6] reported an inversion temperature of about 250°C which agrees with the present work. Their drops were small in size and the free stream Reynolds number was about 100. Hence, the flow was laminar over most of the drop surface with a trailing laminar vortex street. It can be shown [9] that for this situation, the heat transfer coefficient h has a similar trend as given by equation (35). The agreement in the value of the inversion temperature shows that the present model is plausible.

Finally, the variable-property results are compared with the results using the one-third rule. In Fig. 5, the evaporation rates of water for both variable and constant properties are plotted versus the free stream temperature for the $m_{a\infty} = 1$ and $m_{a\infty} = 0$ cases. The results using the one-third rule agree well with the variable-property results, except when the free stream is mostly steam at high temperature. It appears that the one-third rule approximation works very well, even at very high temperature, when the free stream is mostly air. However, when there is a significant amount of steam in the free stream, the one-third rule does not apply for temperatures above 300°C.

CONCLUSIONS

The evaporation of water into a laminar stream of air, superheated steam or a mixture of the two is analyzed. Numerical results are obtained both for variable properties and for constant properties using the onethird rule. The following conclusions can be made:

(1) For the same mass flux of the free stream, and at low free stream temperatures, water evaporates faster



FIG. 5. The comparison between one-third rule and variable-property results.

in air than in humid air and in superheated steam. However, the trend is reversed at high free stream temperatures. Also, the inversion temperature, as defined, is around 250°C.

(2) The existence of the inversion temperature can be explained by the combined effects of higher heat transfer coefficients for steam flows and the interfacial temperature depression by the presence of air.

(3) The constant-property solutions using the onethird rule agree well with the variable-property results except when the free stream is composed of mostly steam at high temperatures.

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EVAPORATION DE L'EAU DANS UN ECOULEMENT LAMINAIRE D'AIR ET DE VAPEUR SURCHAUFFEE

Résumé ---- On étudie numériquement l'évaporation de l'eau dans l'écoulement laminaire d'air, d'air humide et de vapeur surchauffée. Des résultats sont obtenus pour des propriétés variables ou constantes utilisant la règle 1/3. On montre qu'au-dessous d'une certaine température de l'écoulement libre, appelée température d'inversion, l'évaporation de l'eau décroît quand l'humidité de l'air augmente ; et au-dessus de la température d'inversion, l'évaporation augmente quand l'humidité de l'air augmente. L'approximation propriétés constantes utilisant la règle 1/3 donne des résultats précis pour le débit d'évaporation de l'eau, excepté quand l'écoulement libre est principalement de la vapeur à haute température.

VERDUNSTUNG VON WASSER IN EINEN LAMINAREN STROM VON LUFT UND **ÜBERHITZTEM DAMPF**

Zusammenfassung – Zur Untersuchung der Verdunstung von Wasser in einen laminaren Strom von Luft. feuchter Luft und überhitztem Dampf wurde eine numerische Untersuchung durchgeführt. Es wurden Ergebnisse sowohl für variable Stoffeigenschaften als auch für konstante Stoffeigenschaften unter Verwendung der 1/3-Regel erhalten. Es zeigte sich, daß unterhalb einer bestimmten Temperatur der freien Strömung, die Inversionstemperatur genannt wird, die Verdunstung des Wassers mit zunehmender Luftfeuchtigkeit abnimmt. Overhalb der Inversionstemperatur nimmt die Wasserverdunstung mit zunehmender Luftfeuchtigkeit zu. Die Näherungslösung mit konstanten Stoffeigenschaften unter Verwendung der 1/3-Regel führt zu genauen Ergebnissen für die Verdunstungsrate des Wassers, ausgenommen, daß die freie Strömung überwiegend Dampf bei hoher Temperatur enthält.

ИСПАРЕНИЕ ВОДЫ В ЛАМИНАРНЫЕ ПОТОКИ ВОЗДУХА И ПЕРЕГРЕТОГО ПАРА

Проведено численное исследование испарения воды в ламинарные потоки воздуха. Аннотация влажного воздуха и перегретого пара. Результаты получены как для переменных, так и постоянных свойств потока с помощью правила 1/3. Показано, что ниже некоторой температуры свободного потока, называемой температурой инверсии, испарение воды уменьшается с ростом влажности воздуха, в то время как выше температуры инверсии оно возрастает. Использование приближения постоянных свойств потока и правила 1/3 позволяет получить точные результаты для скорости испарения воды за исключением случая, когда свободный поток в основном представляет собой нар при высокой температуре.